Cal NERDS Math Vault: Math 1A Calculus Practice Exam

This exam was created in hopes of giving incoming freshmen and transfer students a sense of what college math (1st semester calculus) at UC Berkeley will be like. This practice exam is to help you prepare for UC Berkeley's Math Courses. If you are unable to complete the exam in the allotted 3 hour time, that is okay. Feel free to continue working on it and check the answers when you are done. If you are using it as a practice test, use the following instructions. Otherwise, your time limit is unlimited!

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Show **all** your work! Take about 3 hours to complete this exam. Try to keep a reasonable pace throughout the exam so you can attempt almost every problem. Skip any problems that you find yourself spending more than 4 minutes on without having made significant progress.

1. Consider the function $f(x) = \sqrt{10 - x} + 3$

   (a) Describe in words how to obtain the graph for $f(x)$ from the graph of $\sqrt{x}$.

   (b) State the domain and range.

(c) Find $f^{-1}(x)$ and find the domain and range for $f^{-1}(x)$.

2. Solve for x: $\log_2(x) + \log_2(2x) = 2$

3. Evaluate the following limits. If a limit is ∞ or $-\infty$, please say so. Make sure you to show your work and justify your answers.

   (a) $\lim_{x\to 2}(x^2 - 3x + 2)$

   (b) $\lim_{x\to 2} \frac{x^2 - 4}{x - 2}$

(c) $\lim_{x \to 0} \frac{|x-2|-2}{x}$

(d) $\lim_{x \to 0} x \sin(\ln(x^4))$

(e) $\lim_{x \to -\infty} (\sqrt{x^2 + 3x} - x)$

(f) $\lim_{x \to \infty} (\sqrt{x^2 + 3x} - x)$

(g) $\lim_{x \to 0} \frac{x^2 e^{3x}}{(\tan x)^2}$

4. Find the largest value of δ such that is $0 < |x-2| < \delta$, then $|\frac{1}{x} - \frac{1}{2}| < \frac{1}{4}$. What does this limit represent?

5. Given that $\epsilon = \frac{1}{2}$ find the largest $\delta > 0$ to prove that the given limit $\frac{x}{x-1} = 2$

6. Find the first three derivatives of the following functions. Simplify your answers as much as possible. Show all your work.

   (a) $f(x) = x^2 \cos(x)$

(b) $f(x) = \frac{x^2-3}{\sqrt{9x-5}}$

(c) $f(x) = \displaystyle\int_{x^2}^{2} \frac{\cos(t)}{t}\, dt$

(d) $f(x) = \frac{\sin(5x)}{x}$

(e) $5x + 4xy^2 = 3y + 15$. Specify what is $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$. No need to calculate $\frac{d^3y}{dx^3}$.

(f) $x^2 - \cos(y) = y^3 + 5x - 1$. Specify what is $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$. No need to calculate $\frac{d^3y}{dx^3}$.

(g) $f(x) = \tan^{-1}(x)$

(h) $f(x) = \sin^{-1}(x)$

7. Suppose you have 100mg of a radioactive substance that decays at a rate of 0.02 mg per day. Find an equation for the amount of substance left after t days.

8. Find the equation of the line tangent to the graph of $f(x) = \frac{1}{x}$ at $x = 8$.

9. The sum of two non-negative numbers is 10

   (a) What is the minimum sum of their squares?

(b) What are the two numbers that minimize the sum of their squares?

(c) What is the maximum sum of their squares?

10. Approximate the area under $f(x) = \sqrt{x+1}$ with four equal subintervals for x-values on the interval $0 \leq x \leq 8$. Use the left Riemann sum, right Riemann sum, midpoint Riemann sum, and trapezoidal sum.

11. Integrate the following functions.

    (a) $f(x) = \sec^2 x$

(b) $g(x) = 11 + \frac{5}{x}$

(c) $h(x) = \frac{1}{2\sqrt{x}} - e^x + \cos(x)$

(d) $p(x) = \frac{7e^{x^5}\cos(7x) - 5x^4 e^{x^5}\sin(7x)}{e^{2x^5}}$

(e) The acceleration of a particle moving along the x-axis is given by $a(t) = e^t - 10t$ for time $0 \leq t \leq 14$. The particle's initial velocity is 13 and its initial position is -3.

i. What is the particle's velocity function? Use it to find $v(2)$. What is the particle's position function? Use it to find its position at time $t = 1$.

ii. What was the particle moving to the left or the right at time $t = 2$? Explain your reasoning.

iii. Was the particle moving toward or away from the origin (from time $t = 0$) at time $t = 3$? Give a reason for your answer?

(f) $f(x) = x^2 \cos(5x^3)$

(g) $g(x) = \frac{1}{x^2} \sec(\frac{3}{x}) \tan(\frac{3}{x})$

12. Find the average value of $f(x) = \sqrt{1 - x^2}$ on the interval $[-1, 1]$

13. Consider the two functions $f(x) = x^2$ and $g(x) = 8 - x^2$. Find the area bound by these two curves.

14. Set-up and integral for its volume, and then evaluate the integral. Sketching out the graph might help. Use disks.

    (a) The region R enclosed by $y = 12 - 3x$ and $y = 0$ on $[0, 4]$ is revolved about the x-axis.

    (b) The region R is enclosed by $y = \frac{x}{6}$, $y = 1$, $x = 0$ is revolved about the y-axis.

15. Set-up and integral for its volume, and then evaluate the integral. Sketching out the graph might help. Use washers.

    (a) The region R enclosed by $y = x^2$ and $y = x^5$ is revolved about the x-axis.

    (b) The region R is enclosed by $y = 9 - x^2$ and $y = 9 - 3x$ is revolved about the y-axis.

1 Bonus Questions

1. Use Newton's method with initial approximation $x_1 = 0$ and two-steps to approximate a solution of the equation $\ln(x+1) = 1/2$. Show all your work.

2. Express $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sin(3+in^{-1})}{n}$ as a definite integral. Do not calculate the numerical value of that integral.

3. Let $f(x) = (x^2 + x + 1)e^{-x}$. Find the critical points, maximum, minimum of f over $[0, 2]$. State where these values are attained.

4. Take the derivative of $f(x) = \frac{\ln(x+2)}{x^2-4x-3}$.